TECHNO INDIA GROUP PUBLIC SCHOOL

MOCK TEST **CLASS-XII**

MATHEMATICS

SET–4

Code: 041

Time : 3 hours F.M.: 80

General Instructions:

Read the following instructions very carefully and strictly follow them:

1. This Question paper contains **38** questions. All questions are compulsory.
2. This Question paper is divided into **five** sections–**A, B, C, D** and **E**.
3. In **Section A**, Questions no. **1** to **18** are **multiple choice questions (MCQs)** with only one correct option and Questions no. **19** and **20** are **Assertion-Reason based** questions of **1 mark each**.
4. In **Section B**, Questions no. **21** to **25** are **Very Short Answer (VSA)–type** questions, carrying **2 marks each**.
5. In **Section C,** Questions no. **26** to **31** are **Short Answer (SA)–type** questions, carrying **3 marks each**.
6. In **Section D,** Questions no. **32** to **35** are **Long Answer (LA)–type** questions, carrying **5 marks each**.
7. In **Section E,** Questions no. **36** to **38** are **Case study-based questions**, carrying **4 marks each.**
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E .
9. Use of calculator is not allowed.

SECTION A

This section comprises of multiple-choice questions (MCQs) of 1 mark each. Select the correct option (Question 1 - Question 18)

1.	If $\alpha = \tan^{-1}\left(\tan\frac{5\pi}{4}\right)$, $\beta = \tan^{-1}\left(-\tan\frac{2\pi}{3}\right)$, then Ⓐ $4\alpha = 3\beta$ Ⓑ $3\alpha = 4\beta$ Ⓒ $\alpha - \beta = \frac{7\pi}{12}$ Ⓓ none of these	[1]
2.	If $A = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$ and $A^2 - xA + yI = 0$, then (x, y) is Ⓐ (3, 7) Ⓑ (9, 14) Ⓒ (5, 14) Ⓓ (3, 14)	[1]
3.	If $A\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$, then A = ? Ⓐ $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ Ⓑ $\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$ Ⓒ $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ Ⓓ none of these	[1]

4.	If A and B are 2 × 2 square matrices and $A + B = \begin{bmatrix} 4 & -3 \\ 1 & 6 \end{bmatrix}$ and $A-B = \begin{bmatrix} -2 & -1 \\ 5 & 2 \end{bmatrix}$, then AB = ? Ⓐ $\begin{bmatrix} -7 & 5 \\ 1 & -5 \end{bmatrix}$ Ⓑ $\begin{bmatrix} 7 & -5 \\ 1 & 5 \end{bmatrix}$ Ⓒ $\begin{bmatrix} 7 & -1 \\ 5 & -5 \end{bmatrix}$ Ⓓ $\begin{bmatrix} 7 & -1 \\ -5 & 5 \end{bmatrix}$	[1]
5.	Let $f(t) = \begin{vmatrix} \cos t & t & 1 \\ 2\sin t & t & 2t \\ \sin t & t & t \end{vmatrix}$, then $\lim_{t\to 0} \frac{f(t)}{t^2}$ is equal to Ⓐ 1 Ⓑ 0 Ⓒ –1 Ⓓ 2	[1]
6.	There are two values of a which makes determinant, $\Delta = \begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86$, then sum of these number is Ⓐ 4 Ⓑ 5 Ⓒ –4 Ⓓ 9	[1]
7.	If $f(x) = \begin{cases} \frac{\sin 5x}{x^2+2x}, & x \neq 0 \\ k + \frac{1}{2}, & x = 0 \end{cases}$ is continuous at x = 0, then the value of k is Ⓐ 1 Ⓑ –2 Ⓒ 2 Ⓓ $\frac{1}{2}$	[1]
8.	If $y = \log\left(\frac{1-x^2}{1+x^2}\right)$, then $\frac{dy}{dx}$ is equal to Ⓐ $\frac{4x^3}{1-x^4}$ Ⓑ $\frac{-4x}{1-x^4}$ Ⓒ $\frac{1}{4-x^4}$ Ⓓ $\frac{-4x^3}{1-x^4}$	[1]
9.	The function $f(x) = 2x^3 - 15x^2 + 36x + 6$ is strictly decreasing on Ⓐ $(-\infty, 2] \cup [3, \infty)$ Ⓑ $[2, 3]$ Ⓒ $(2, 3)$ Ⓓ $(-\infty, 2) \cup (3, \infty)$	[1]
10.	The solution of the differential equation $\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$ is Ⓐ $x^2 - y^2 = cx$ Ⓑ $x^2 + y^2 = cy$ Ⓒ $x^2 + y^2 = cx$ Ⓓ none of these	[1]
11.	$\int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x\cos^2 x} dx$, equals Ⓐ $\sin^2 x + c$ Ⓑ $\sin 2x + c$ Ⓒ $-\frac{1}{2}\sin 2x + c$ Ⓓ $-\frac{1}{2}\cos 4x + c$	[1]
12.	$\int_0^{\frac{\pi}{2}} e^x \left(\frac{1+\sin x}{1+\cos x}\right) dx = ?$ Ⓐ 0 Ⓑ $\frac{\pi}{4}$ Ⓒ $e^{\frac{\pi}{2}}$ Ⓓ $e^{\frac{\pi}{2}} - 1$	[1]
13.	If $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$, then angle between $\left(\vec{a} + \vec{b}\right)$ and $\left(\vec{a} - \vec{b}\right)$ is Ⓐ $\frac{\pi}{3}$ Ⓑ $\frac{\pi}{4}$ Ⓒ $\frac{\pi}{2}$ Ⓓ $\frac{2\pi}{3}$	[1]
14.	If the vectors $\vec{a} = 4\hat{i} + 11\hat{j} + x\hat{k}$, $\vec{b} = 7\hat{i} + 2\hat{j} + 6\hat{k}$ and $\vec{c} = \hat{i} + 5\hat{j} + 4\hat{k}$ are coplanar, then value of x Ⓐ 0 Ⓑ 38 Ⓒ 10 Ⓓ 20	[1]
15.	If $\left\|\vec{a} \times \vec{b}\right\| = 4, \left\|\vec{a} . \vec{b}\right\| = 2$, then $\left\|\vec{a}\right\|^2 \left\|\vec{b}\right\|^2$ is Ⓐ 6 Ⓑ 2 Ⓒ 20 Ⓓ 8	[1]

16. Corner points of the feasible region for an LPP are (0, 2), (3,0), (6, 0), (6, 8) and (0, 5). **[1]**

Let F = 4x + 6y be the objective function. The minimum value of F occurs at

Ⓐ only (0, 2)

Ⓑ only (3, 0)

Ⓒ the mid-point of the line segment joining the points (0, 2) and (3, 0)

Ⓓ any point on the line segment joining the points (0, 2) and (3, 0)

17. A set of values of decision variables that satisfies he liear constraints and non-negativity conditions of an L.P.P. is called its **[1]**

Ⓐ Unbounded solution Ⓑ Optimum solution

Ⓒ Feasible solution Ⓓ None of these

18. Eight coins are thrown simultaneously. Find the probability of getting atleast 6 heads. **[1]**

Ⓐ $\frac{31}{128}$ Ⓑ $\frac{37}{256}$ Ⓒ $\frac{37}{128}$ Ⓓ $\frac{31}{256}$

Assertion Reason based Questions :

Directions: Questions numbers 19 and 20 are Assertion–Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is not the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

19. **Assertion (A) :** Range of $\cot^{-1}x$ is $(0, \pi)$ **[1]**

Reason (R) : Domain of $\tan^{-1}x$ is R.

Ⓐ A Ⓑ B Ⓒ C Ⓓ D

20. **Assertion (A) :** The value of $\hat{i}.\left(\hat{j} \times \hat{k}\right) + \hat{j}.\left(\hat{j} \times \hat{k}\right) + \hat{k}.\left(\hat{i} \times \hat{j}\right)$ is 1. **[1]**

Reason (R) : Since, $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 0$

Ⓐ A Ⓑ B Ⓒ C Ⓓ D

SECTION B

This section comprises of 5 very short answer (VSA) type questions of 2 marks each.

21A Find the value of $\sin^{-1}\left(\cos\left(\frac{33\pi}{5}\right)\right)$ **[2]**

OR

21B Express $\tan^{-1}\left(\frac{\cos x}{1 - \sin x}\right), -\frac{\pi}{2} < x < \frac{\pi}{2}$ in the simplest form. **[2]**

22. If $y = 2\tan^{-1}\sqrt{\frac{x - a}{b - x}}$ and a < x < b, then show that $\left(\frac{dy}{dx}\right)^2 + \frac{1}{(x - a)(x - b)} = 0$. **[2]**

23A	Find the area of the region included between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$, where $a > 0$. **OR**	[2]
23B	Find the area of the region $\{x, y) : x^2 + y^2 \leq 1 \leq x + y\}$.	[2]
24.	Prove that the function f given by $f(x) = \lvert x - 1 \rvert, x \in R$ is not differentiable at $x = 1$.	[2]
25.	Show that the vectors $2\hat{i} - \hat{j} + \hat{k},\ \hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$, form the vertices of a right angled triangle.	[2]

SECTION C

This section comprises of 6 short answer (SA) type questions of 3 marks each.

26A	Differentiate the following functions w.r.t.x : $\tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right), -\frac{\pi}{2} < x < \frac{\pi}{2}, \frac{a}{b}\tan x > -1$ **OR**	[3]
26B	If $y = \frac{2}{\sqrt{a^2 - b^2}}\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right)$, prove that $\frac{dy}{dx} = \frac{1}{a + b\cos x}$; $a > b > 0$.	[3]
27.	A spherical ball of salt is dissolving in water in such a manner that the rate of decrease of the volume at any instant is proportional to the surface area. Prove that the radius is decreasing at a constant rate.	[3]
28A	Make a rough sketch of the region given below and find its area using integration $\{(x, y) : 0 \leq y \leq x^2 + 3; 0 \leq y \leq 2x + 3, 0 \leq x \leq 3\}$. **OR**	[3]
28B	Find the area bounded by the curve $y = \cos x$ between $x = 0$ and $x = 2\pi$.	[3]
29A	Find the equation of a line passing through the point P(2, –1, 3) and perpendicular to the lines : $\vec{r} = \left(\hat{i} + \hat{j} - \hat{k}\right) + \lambda\left(2\hat{i} - 2\hat{j} + \hat{k}\right)$ and $\vec{r} = \left(2\hat{i} - \hat{j} - 3\hat{k}\right) + \mu\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$ **OR**	[3]
29B	Find the shortest distance between the lines whose vector equations are : $\vec{r} = \left(\hat{i} + \hat{j}\right) + \lambda\left(2\hat{i} - \hat{j} + \hat{k}\right)$ and $\vec{r} = \left(2\hat{i} + \hat{j} - \hat{k}\right) + \mu\left(3\hat{i} - 5\hat{j} + 2\hat{k}\right)$	[3]
30.	Solve the following Linear Programming Problem graphically : Maximise $z = 8x + 9y$ subject to the constraints : $2x + 3y \leq 6$, $3x - 2y \leq 6$, $y \leq 1$; $x, y \geq 0$.	[3]
31.	In a hostel, 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspaper. A student is selected at random. (a) Find the probability that the student reads neither Hindi nor English newspaper. (b) If she reads Hindi newspaper, find the probability that she reads English newspaper. (c) If she reads English newspaper, find the probability that she reads Hindi newspaper.	[3]

SECTION D

This section comprises of 4 long answer (LA) type questions of 5 marks each.

32.	If $A = \begin{vmatrix} 3 & 1 & 2 \\ 3 & 2 & -3 \\ 2 & 0 & -1 \end{vmatrix}$, find A^{-1}. Use A^{-1} to solve the following system of equations : $3x + 3y + 2z = 1$; $x + 2y = 4$; $2x - 3y - z = 5$.	[5]

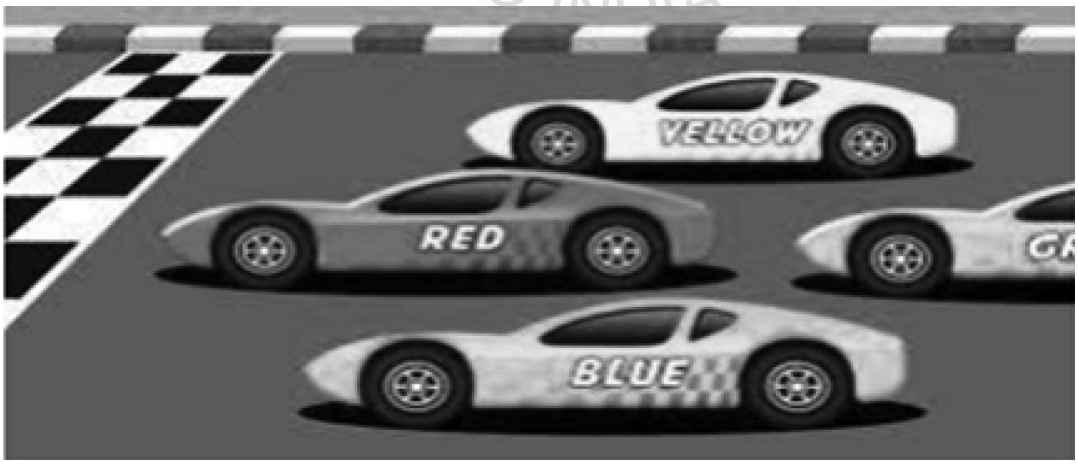
33A	Evaluate $\int (x-3)\sqrt{x^2+3x-18}\ dx$.	[5]
	OR	
33B	$\int_{-5}^{0} (\lvert x\rvert + \lvert x+2\rvert + \lvert x+5\rvert)dx$	[5]
34A	Solve the differential equation $x\log x \frac{dy}{dx} + y = \frac{2}{x}\log x$.	[5]
	OR	
34B	Find the particular solution of the differential equation $\left\{x\sin^2\left(\frac{y}{x}\right) - y\right\} dx + x\, dy = 0$, given that $y = \frac{\pi}{4}$, when x = 1.	
35.	Show that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ intersect. Also, find their point of intersection.	[5]

SECTION E

This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has wo subparts of 2 marks each)

Case Study - 1

36. Read the following passage and answer the questions given below.
An organization conducted car race under two different categories boys and girls. Total there are 300 participants. Among all of them 2 from category 1 and 3 from category 2 were selected for the final race. Raju forms two sets B and G for these participants B = $\{b_1, b_2\}$ G = $\{g_1, g_2, g_3\}$ where B represents the set of boys and G set of girls, who were selected for the final race.



I. Raju wishes to form all the relationship possible from B to G. How many such relations are possible ? [1]

II. How many functions can be formed from B to G ? [1]

III. A let R : B → G defined by R = $\{(b_1, g_1), (b_2, g_2)\}$ then what kind of function R is ? [2]

OR

III. B Find the number of surjective functions from B to G. [2]

Cont.. 6

37. **Case Study - 2**

A building contractor undertakes a job to construct 4 flats on a plot along with parking area. Due to strike the probability of many construction workers not being present for the job is 0.65. The probability that many are not present and still the work gets completed on time is 0.35. The probability that work will be completed on time when all workers were present is 0.80.

Let E_1 : Represent the event when many workers were not present for the job.

E_2 : Represent the event when all workers were present and E : represent completing the construction work on time.

Based on the above information, solve the following questiions :

I. What is the probability that all the workers are present for the job? [1]

II. What is the probability that construction will be completed on time ? [1]

III A. What is the probability that many workers are not present given that the constructiion work is completed on time? [2]

OR

III B. What is the probability that all workers were present given that the construction job was completed on time ? [2]

38. **Case Study - 3**

Read the following passage and answer the questions given below :

The shape of a toy is given as $f(x) = 8x^4 - 4x^2 + 3$.

To make the toy beautiful 2 sticks which are perpendicular to each other were placed at a point (4, 5) above the toy. Based on the above information, solve the following questions :

I. Find the maximum value of the function. [2]

II. At which interval will f(x) be decreasing ? [2]